Transformations Review

Stretches:

$y = af(x)$	$\lvert a\rvert > 0$	vertical expansion	for every x value, y expands by a factor of a
	$0 < \lvert a\rvert < 1$	vertical compression	for every x value, y compresses by a factor of a
$y = f(bx)$	$\lvert b\rvert > 0$	horizontal compression	for every y value, x compresses by a factor of $\frac{1}{b}$
	$0 < \lvert b\rvert < 1$	horizontal expansion	for every y value, x expands by a factor of $\frac{1}{b}$

Reflections:

$y = -f(x)$	reflection in the $x-axis$	y values become $-y$
$y = f(-x)$	reflection in the $y-axis$	x values become $-x$
$y = f^{-1}(x)$	reflection in the line $y = x$	(x, y) coordinates become (y, x)

Translations:

$y = f(x)+k$	$k > 0$	vertical translation up	y values become $y+k$
	$k < 0$	vertical translation down	y values become $y-k$
$y = f(x-h)$	$h > 0$	horizontal translation right	x values become $x+h$
	$h < 0$	horizontal translation left	x values become $x-h$

*** $y = f(x-3)$, $h = 3$, x values become $x+3$

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Order of Transformations Given the form : $y = af(b(x-h))+k$

1. Stretches/Reflections

2. Translations

$y = f^{-1}(x)$

For the graph or points of the inverse of a function, (x, y) coordinates switch. When working with equations, switch x and y then solve for y.

$y = \dfrac{1}{f(x)}$

For the reciprocal of a function, every y value becomes $\dfrac{1}{y}$. When $y = 0$, a vertical asymptote occurs.

$y = |f(x)|$

All negative y values become positive. All positive y values stay positive. $y = 0$ remains $y = 0$.

$y = f(|x|)$

For positive x values and $x = 0$, y values remain unchanged. For negative x values, the y values become the same y value as the equivalent positive x value has. Visually, the graph for $x < 0$ mirrors the right side, $x \geq 0$.

Key points to remember:

1. **FACTOR, FACTOR, FACTOR!**

2. $f(x)$ represents a y value

3. Anything affecting the x value is in the $(\ )$

4. Absolute value signs are a type of bracket. Do what is in the brackets first.

5. When changing an equation, replace the affected letter with brackets.
   Example: Translation 2 units right. Replace x with $(x-2)$

Review Questions

1. Point $A(3,-4)$ is on the graph of $y = f(x)$. What point must be on the graph of:

   a) $y = 2f(x-3)+1$

   b) $y-3 = f(2x-4)$

   c) $y = -\frac{1}{4}f(6-x)-5$

   d) $y = \frac{1}{f(x+1)}+3$

   e) $y = \left|\frac{1}{f(3x)}\right|-2$

   f) $y = f^{-1}(x)$

2. Describe how the function relates to $y = f(x)$.

   a) $y = \frac{1}{2}f(x)-4$

   b) $4y = f(x-3)+8$

   c) $y = f(6-2x)$

   d) $y = -f(x)-7$

   e) $y = f^{-1}(x)$

   f) $y = -f(-x)+1$

3. The graph of $y=(x-2)^2-1$ represents $y=f(x)$. What are the coordinates of the point(s) that would be invariant for the following transformations?

a) $y=-f(x)$

b) $y=f(-x)$

c) $y=\frac{1}{2}f(x)$

d) $y=f(4x)$

e) $y=\frac{1}{f(x)}$ (for this transformation, give just one answer of the three possible)

4. Given the following equations, determine the new equation after the following transformations.

a) $y=\sqrt{x}$ a vertical expansion by a factor of 3, then a reflection in the y-axis, then a translation 7 units up

b) $y=|x|$ a horizontal compression by a factor of $\frac{1}{2}$, then a reflection in the x-axis, then a translation 4 units left

c) $y=3x-5$ a reflection in the line y=x, then a translation 2 units right, then a translation 5 units down

d) $8x^2+y^2=2$ a horizontal expansion by a factor of 2, then a vertical compression by a factor of $\frac{1}{3}$.

e) $y=x^4+x^3-2x^2+3$ a reflection in the y-axis, then a translation of 5 units up, then a vertical expansion by a factor of 3

5. Determine the equation of $y = f^{-1}(x)$, the inverse, for the following equations.

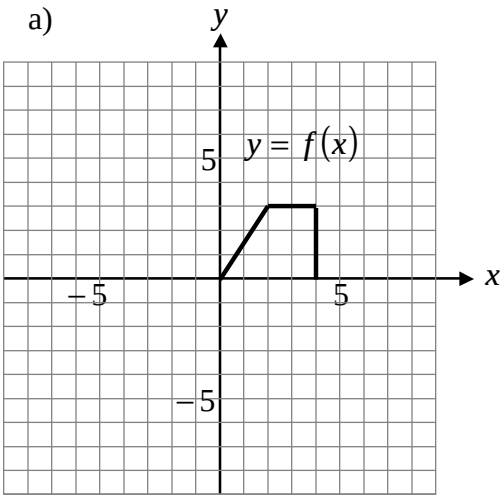
a) $f(x) = \frac{1}{3}x + 2$

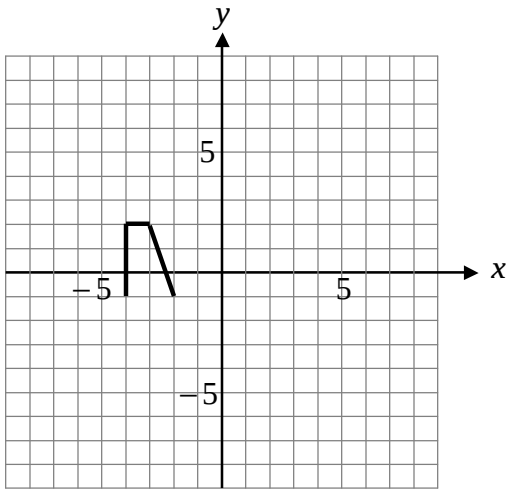
b) $f(x) = 2(x+3)^2 - 4$

c) $f(x) = \frac{1}{x+2}$

6. Given the graphs of $y = f(x)$, determine the equations of the graphs with the transformations shown.

a)





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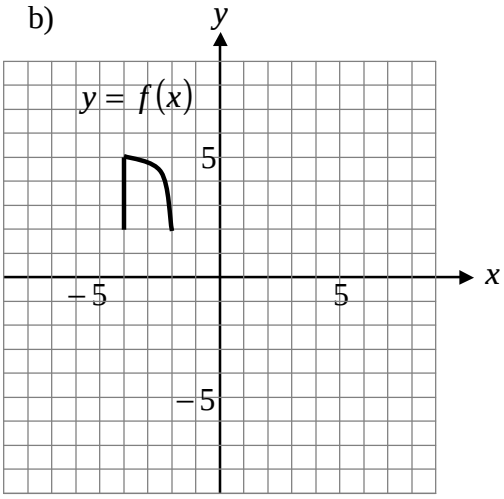

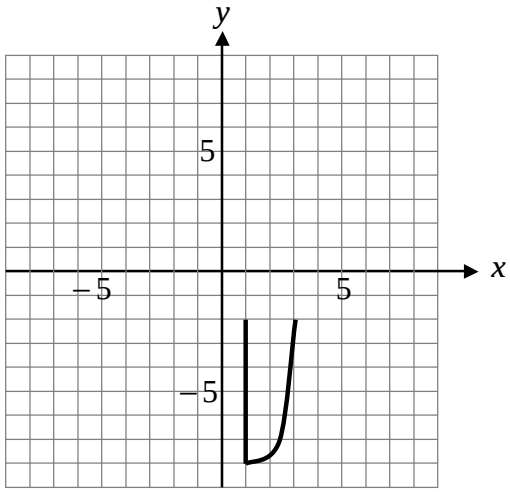

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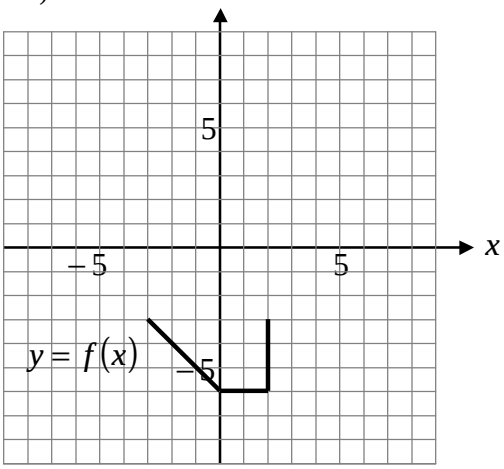
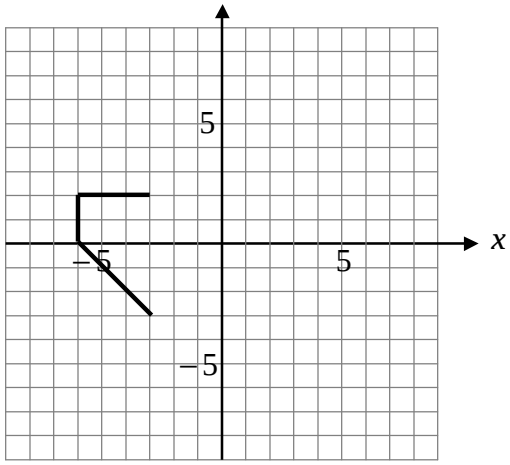
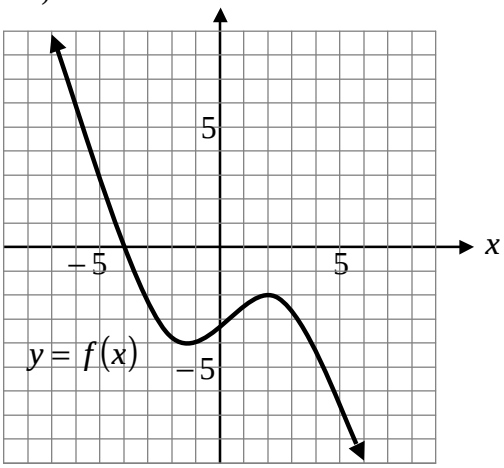
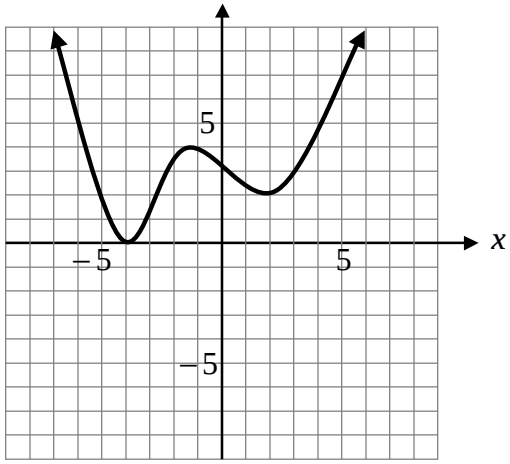


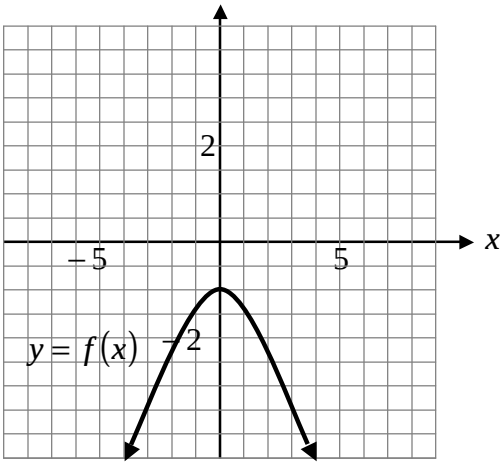


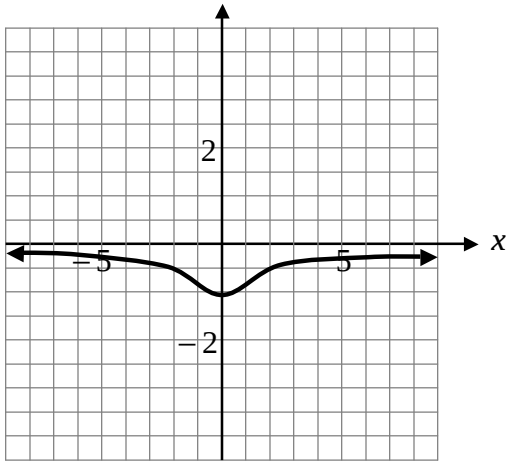
d)





e)





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7. Point $C(a,b)$ is on the graph of $y = f(x)$.

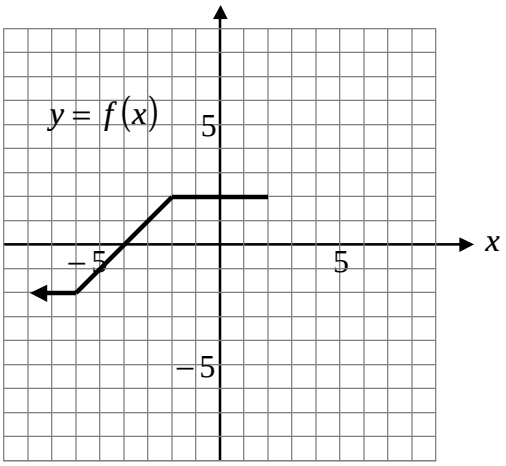
a) What point must be on the graph of $y = -\frac{1}{3} f(x+4)$?

b) What point must be on the graph of $y = \frac{1}{f(x-2)} + 3$?

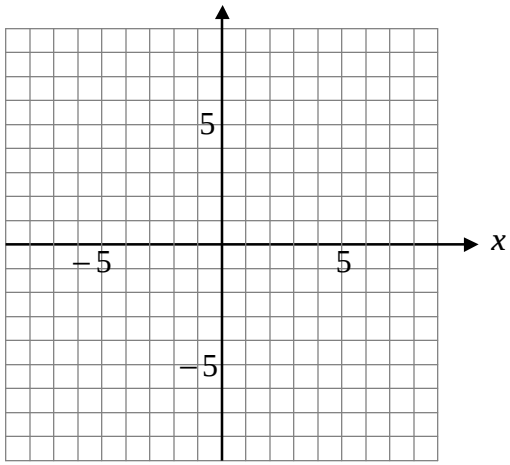
c) If point C is the vertex of a parabola that opens up, what is the domain and range of $y = f^{-1}(x)$?

d) If point C is the vertex of a parabola that opens up, and $a > 0$, $b < 0$, what is the domain and range of $y = |f(x)|$?

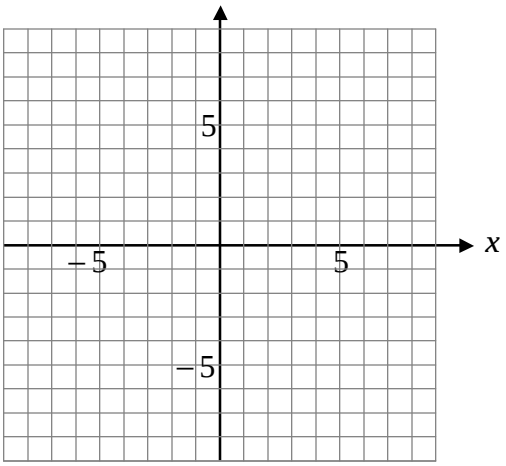
8. Given the graph of $y = f(x)$, sketch the following graphs:



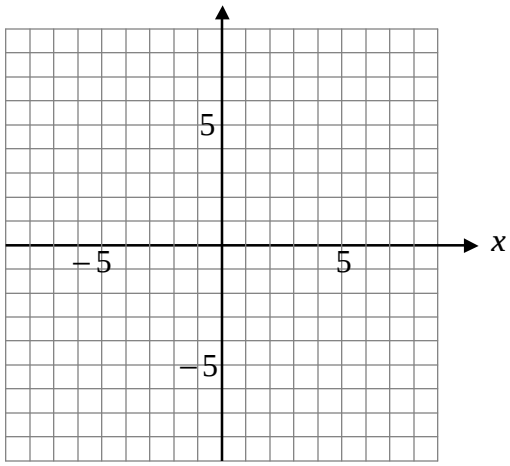
a) $y = f(x-2)-3$



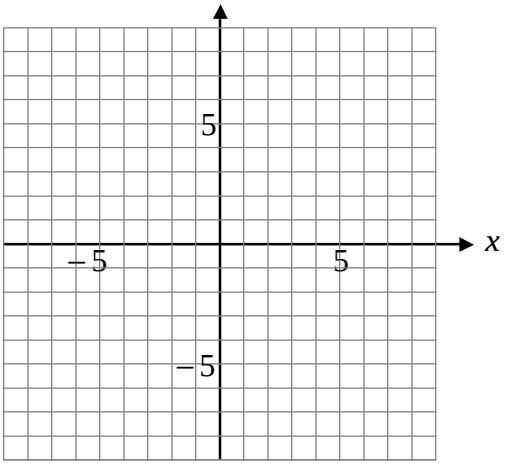
b) $y = f^{-1}(x)$



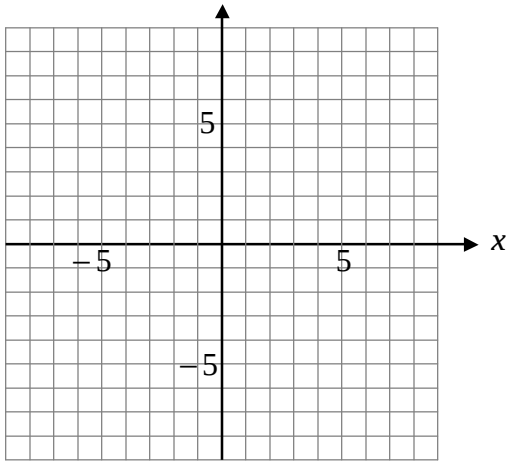
c) $y = |f(x)|$



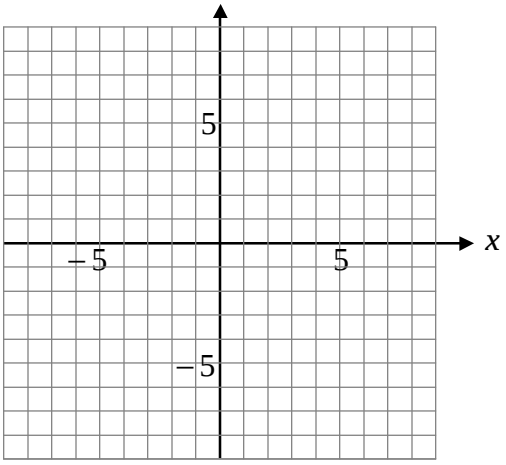
d) $y = f(|x|)$



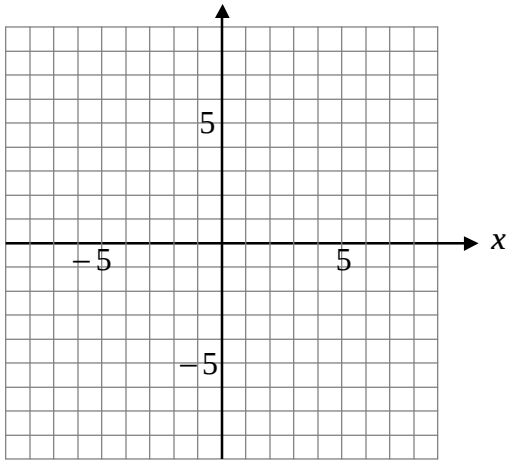
e) $y = \dfrac{1}{f(x)}$



f) $y = 2f(-x)$



g) $y = f(2x+2)$



Solutions

1. a) $(6,-7)$ b) $\left(\frac{7}{2},-1\right)$ c) $(3,-4)$ d) $\left(2,\frac{11}{4}\right)$ e) $(1,2)$ f) $(-4,3)$

2a) Vertical compression by a factor of $\frac{1}{2}$, then a vertical translation 4 units down.

b) Vertical compression by a factor of $\frac{1}{4}$, then a horizontal translation 3 units right and a vertical translation 8 units up.

c) Reflection in the $y-axis$ and a horizontal compression by a factor of $\frac{1}{2}$, then a horizontal translation 3 units right.

d) Reflection in the $x-axis$, then a vertical translation 7 units down.

e) Reflection in the line $y=x$.

f) Reflection in the $x-axis$ and a reflection in the $y-axis$, then a vertical translation 1 unit up.

3. a) $(1,0),(3,0)$ b) $(0,3)$ c) $(1,0),(3,0)$ d) $(0,3)$ e) $(2,-1)$

4. a) $y=3\sqrt{-x}+7$ b) $y=-|2x+8|$ c) $y=\frac{1}{3}x-4$ d) $2x^2+9y^2=2$

e) $y=3x^4-3x^3-6x^2+24$

5. a) $y=3x-6$ b) $y=\pm\sqrt{\frac{x+4}{2}}-3$ c) $y=\frac{1}{x}-2$

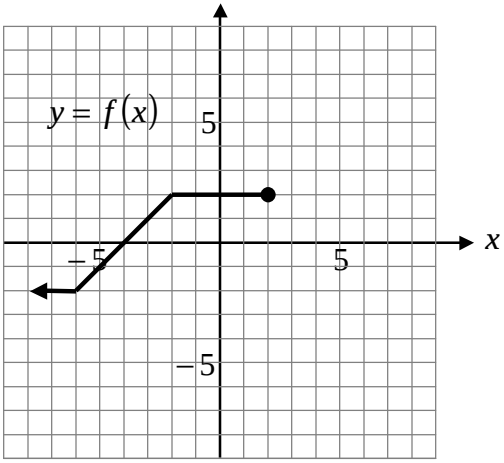
6. a) $y=f(-2x-4)-1$ b) $y=-2f(x-5)+2$ c) $y=f^{-1}(x)$

d) $y=|f(x)|$ e) $y=\frac{1}{f(x)}$

7. a) $\left(a-4,-\frac{b}{3}\right)$ b) $\left(a+2,\frac{1}{b}+3\right)$ c) $D: x\geq b,\ R:$ all real numbers

d) $D:$ all real numbers, $R: y\geq 0$

8.



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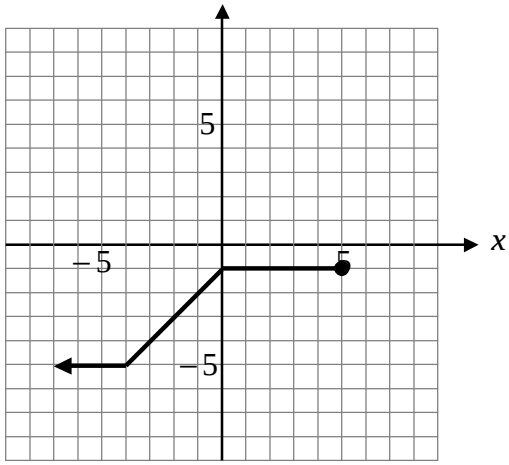
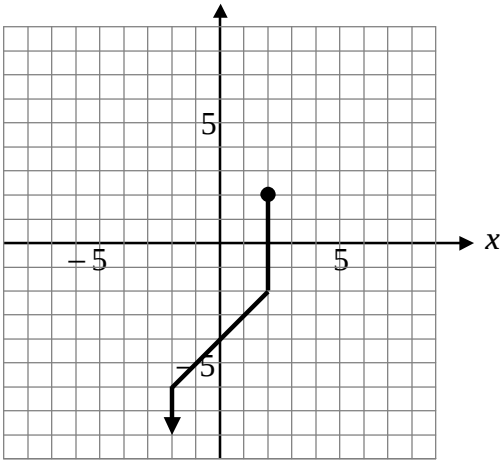
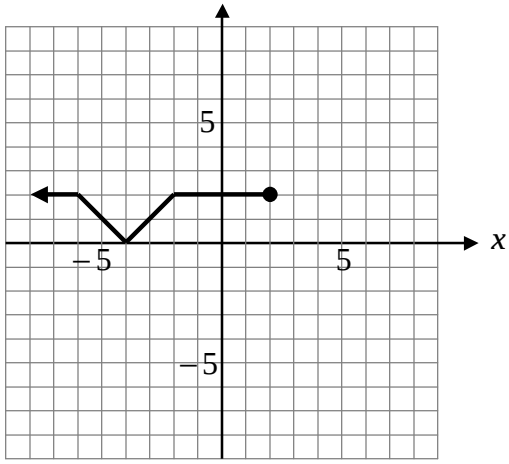


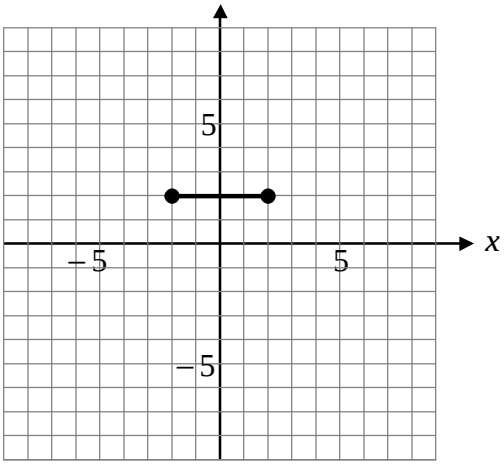
b) $y = f^{-1}(x)$



c) $y = |f(x)|$



d) $y = f(|x|)$



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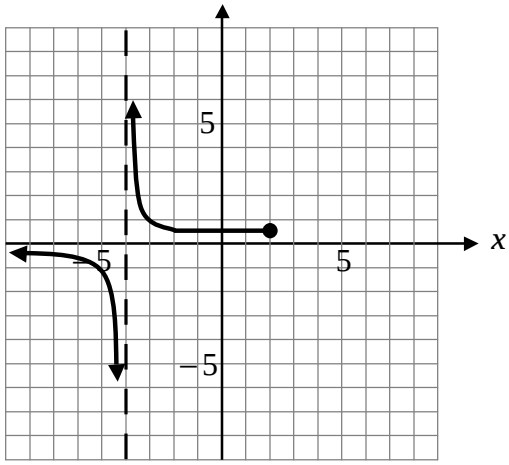

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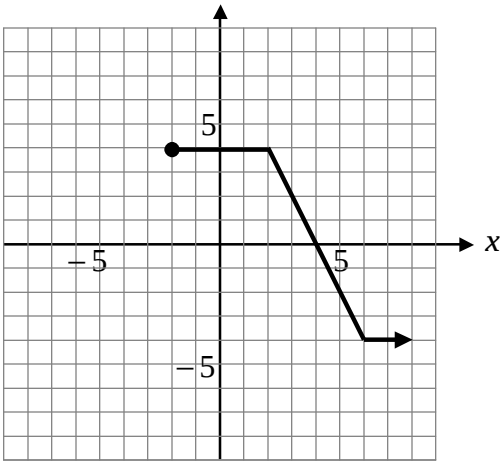

g) $y = f(2x+2)$